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An Efficient Boundary Handling with a Modified Density Calculation for SPH: Supplementary Material

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This supplementary document describes a derivation of Algorithm 1 in the paper *An Efficient Boundary Handling with a Modified Density Calculation for SPH* presented at Pacific Graphics 2015.

1. Derivation of Algorithm 1

In order to derive Algorithm 1, we consider three pattern shown in Figure 1.

Concave shared edges

Figure 1(a) shows a case that all shared edges in an effective sphere are concave. We assume that $\gamma_1, \gamma_2, \gamma 3$ are γ values for planes ①, ②, ③ respectively. Also, $\gamma'_{12}, \gamma'_{23}$, which are integral of $W(r,h)$ in overlapping region with two planes (see main text), have been calculated and stored in shared edges A,B. From Figure 1(a), we can define a complementary set of γ for an effective sphere as follows.

$$\gamma' = (1-\gamma_1) + (1-\gamma_2) + (1-\gamma_3) - \gamma'_{12} - \gamma'_{23} \tag{1}$$

Equation (1) is also derived from a sum of γ' for all shared edges.

The shared edge A is composed of two plane ①, ②, therefore γ' for the edge A can be the following.

$$\gamma'_A = (1-\gamma_1) + (1-\gamma_2) - \gamma'_{12} \tag{2}$$

We can construct similar equation for the shared edge B. However, it is necessary to subtract a region framed by blue dash line in Figure 1(a).

$$\gamma'_B = (1-\gamma_3) - \gamma'_{23} \tag{3}$$

Hence Equation (1) is also defined by summing up Equation (2) and Equation (3).

From Equation (2) and Equation (3), γ' at a concave shared edge with two planes (polygons) i, j is calculated by, (1) subtracting γ'_{ij} from γ', (2) adding $(1-\gamma_k)$ to γ' if another shared edges included in plane k is an unprocessed concave edge or the plane k does not include another shared edge ($k = i$ or $k = j$). These procedures correspond to line 7-16 in Algorithm 1.

Convex shared edges

Figure 1(b) shows a case that all shared edges in an effective sphere are convex. From Figure 1(a), we can define a complementary set of γ as follows.

$$\gamma' = \gamma'_{12} - ((1-\gamma_2) - \gamma'_{23}) \tag{4}$$

Equation (4) is also derived from a sum of γ' for all shared edges.

The shared edge A is composed of two plane ①, ②, therefore γ' for the edge A can be the following.

$$\gamma'_A = \gamma'_{12} \tag{5}$$

The shared edge B is connected with the processed shared edge A via the plane ② and its region includes a region already used for the shared edge A. Therefore, we simply subtract γ for the region framed by red dash line in Figure 1(b).

$$\gamma'_B = \gamma'_{23} - (1-\gamma_2) \tag{6}$$

Hence Equation (4) is also defined by summing up Equation (5) and Equation (6).

From Equation (5) and Equation (6), γ' at a convex shared edge with two planes (polygons) i, j is calculated by, (1) adding γ'_{ij} to γ', (2) subtracting $(1-\gamma_k)$ from γ' if another shared edges included in plane k is an unprocessed convex edge ($k = i$ or $k = j$). These procedures correspond to line 21-28 in Algorithm 1.

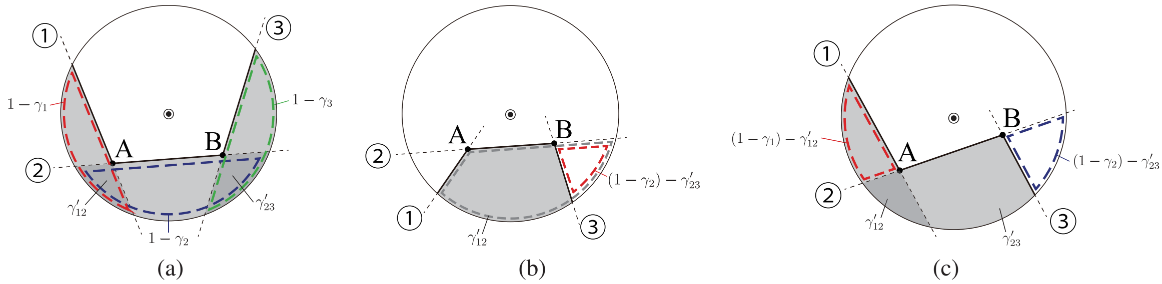

Figure 1: *Conbination of shared edges.*

Algorithm 1 γ calculation.

$\gamma' = 0$
for each plane i without shared edge **do**
  $\gamma' += 1-\gamma_i$
end for
for each shared edge e formed by planes i and j **do**
  if edge is concave **then**
    $\gamma' -= \gamma'_{ij}$
    for $p \in i, j$ **do**
      if plane p includes another shared edge **then**
        if another shared edge is convex
            or unprocessed concave edge **then**
          $\gamma' += 1-\gamma_p$
        end if
      else
        $\gamma' += 1-\gamma_p$
      end if
    end for
    if edge includes shared vertex v **then**
      $\gamma' -= f_d(\tilde{W})\gamma'_v$
    end if
  else
    $\gamma' += \gamma'_{ij}$
    for $p \in i, j$ **do**
      if plane p includes another shared edge **then**
        if another shared edge is concave
            or unprocessed convex edge **then**
          $\gamma' -= 1-\gamma_p$
        end if
      end if
    end for
  end if
  mark e as processed
end for
for each shared vertex v **do**
  $\gamma' += f_d(\tilde{W})\gamma'_v$
end for
return $1-\gamma'$

Mixture of concave and convex edge

Figure 1(c) shows a case that both concave and convex edges are included. In this case, a complementary set of γ is calculated as follows.

$$\gamma' = (1-\gamma_1) - \gamma'_{12} + \gamma'_{23} \tag{7}$$

Equation (7) is also derived from a sum of γ' for all shared edges. We consider two patterns regarding to calculation order, convex edge A $\rightarrow$ concave edge B and concave edge B $\rightarrow$ convex edge A.

At first, we show a procedure in case of A $\rightarrow$ B. The shared edge A is convex. Thus we can re-use Equation (2).

$$\gamma'_A = (1-\gamma_1) + (1-\gamma_2) - \gamma'_{12} \tag{8}$$

The shared edge B is connected with the processed shared edge A via the plane ② and its region includes a region already used for the shared edge A. Therefore, we simply subtract γ for the region framed by blue dash line in Figure 1(c).

$$\gamma'_B = \gamma'_{23} - (1-\gamma_2) \tag{9}$$

We can get Equation (7) by summing up Equation (8) and Equation (9).

In case of B $\rightarrow$ A, we can re-use Equation (5) because the shared edge B is convex.

$$\gamma'_B = \gamma'_{23} \tag{10}$$

The shared edge A is connected with the processed shared edge B. Therefore, we just add γ for the region framed by red dash line in Figure 1(c) which does not included in process for the edge B.

$$\gamma'_A = (1-\gamma_1) - \gamma'_{12} \tag{11}$$

Hence Equation (7) is also defined by summing up Equation (10) and Equation (11) as same as the procedure in case of A$\rightarrow$B.

We have to add some condition to the procedure described in previous two section from Equation (8) and Equation (9). For a convex shared edge, a condition that another shared edge is convex should be appended. For a concave shared edge, a condition that another shared edge is concave should be appended. These conditions correspond to line 10 and 24 in Algorithm 1.