

# PARTICLE-BASED VOLUMETRIC HAIR INTERACTION WITH TURBULENT FLOW

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## ABSTRACT

We present an efficient method of simulating hair interaction with turbulent flow. Hair interaction is one of the critical problems for hair animation, and many methods have been proposed to solve this problem in computer graphics field. We focus on the hair-air interaction including turbulent flow because it drastically affects hair streaming. In this paper, we integrate a wavelet analysis with vorticity confinement to control the turbulent flow. It realizes that users can easily control the hair streaming with only one parameter. Also, our method solves both hair-hair and hair-air interaction with a unified particle-based method that enables fast, natural two-way interaction.

**Keywords:** hair simulation, hair interaction, wind, turbulence

## 1. INTRODUCTION

Simulating the dynamics of hair plays a critical role in character animation of computer graphics because most characters such as humans and animals have the hair. The interaction between each hair strand is necessary to represent physical behaviors like friction effect and to keep its volume; it is a challenging task due to the massive number of strands. To solve this problem, most of the recent research used reduced hair models for real-time computing instead of full hair simulation, and it realized a realistic hair animation. On the other hand, the interaction between hair and air remains an important problem for hair simulation. The air drag force would cause hair velocity damping. Also, the wind including turbulent flow will drastically affect hair behavior. The combination of multiple phenomena makes the simulation more complicated and increases computational cost. Therefore, our goal is to develop fast hair interaction simulation including multiple phenomena: air drag force, turbulent flow, etc.

In this paper, we present an efficient method of simulating hair interaction with the turbulent flow to realize controllable hair streaming. Our approach is to use a unified position-based solver for hair interaction by approximating hair as a continuum fluid that enables natural two-way in-

teraction with the air. We use the Cosserat rod model to make the individual movement of hair strands and then use Position Based Fluids (PBF)[1] for hair-hair and hair-air interactions. Although numerical dissipation damps out turbulent flows, we integrate wavelet analysis with vorticity confinement[2] to control turbulent flow. It enables controlling vortexes in various scale coming from hair streaming by only one user-defined parameter.

Our method is based on the Position Based Dynamics (PBD)[3] framework for its stability and controllability. Also, we accelerated the simulation using the parallel computation by GPU due to the full Lagrangian representation which can divide the problem into particles, making it easy to parallelize.

## 2. RELATED WORKS

### 2.1. Hair Dynamics

A survey of earlier hair simulation methods in computer graphics is available in Ward et al.[4]. Since realistic hair simulation is in common problem in computer graphics, many researchers have modeled the hair as the elastic or inextensible body. In recent research, Umetani et al.[5] integrated the Cosserat rod model into the PBD framework which enables fast and stable simulation. It contains the torsion and non-linear effects of strands resulting in more complex hairstyles like curly and wavy hair by representing the material frame with a ghost point. Kugelstadt et al.[6] enhanced their model that uses a quaternion instead of the ghost point. Some extended method have been proposed, however, this paper use the rod model using the quaternion[6] for hair dynamics because of its simplicity.

### 2.2. Hair Interaction

Hair-hair interaction is one of the important factors for hair animation. Much recent research focused on friction effects[7] and volumetric effects[8]. On the other hand, the methods that treat the hair as a continuum fluid have been proposed[9][10]. We apply this approach for our simulation so that it can realize natural two-way interaction with an external fluid such as air, with a unified solver.

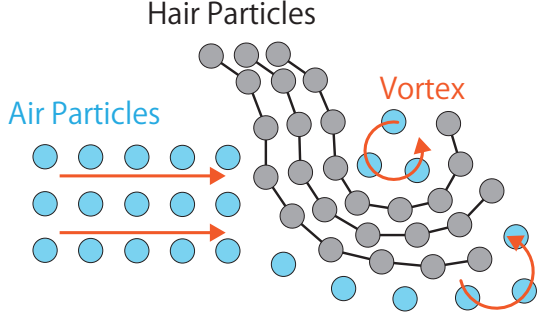


Figure 1: Interaction with turbulent flow

### 3. METHODS

A hair strand fundamentally behaves as an elastic body. We use the elastic rod model using the quaternion[6] to simulate single hair strand dynamics. This method represents bending and twisting property with stable, simple constraints.

In the hair-hair interaction part, we introduce an approach which treats the hair as a continuum fluid[9]. Some studies combined a particle and a grid method[7][10], however, we use a particle-based method for the scalability of computational space. We apply PBF for the fluid simulation because of its incompressibility and stability.

Since we calculate hair-hair interaction as continuum fluid, we just add air particles in the PBF solver to consider hair-air interaction. We do not add a gravity force to the air particles because the air particles only exist in a windy area. To achieve realistic hair animation, considering the turbulent flow that occurs in hair-air interaction is necessary (see Fig.1). We use vorticity confinement for the simulation with the turbulent flow because of its high controllability. To strongly emphasize the turbulence with a specific scale, we use wavelet analysis to estimate the energy of the turbulent that have the scale. Then we apply vorticity confinement according to the energy.

#### 3.1. Hair-Hair Interaction

We use particles instead of cylindrical shape collision detection or edge-edge distance constraint for hair-hair interaction. For accurate hair-hair interaction to simulate volumetric hair, it is necessary to densely distribute many particles and consider the air particles in the space between the hair strands. However, we just mapped the particles on each discretized points of the hair strands for fast computing. We assume that the hair particles represent a part of a hair strand and the air surrounding it. This approach also enables the self-interaction in a single hair strand.

We implemented this using PBF. PBF solves a system of density constraints to enforce fluid incompressibility. The

constraints  $C$  in the particle  $i$  is defined as:

$$C_i(\mathbf{p}_1, \dots, \mathbf{p}_n) = \frac{\rho_i}{\rho_0} - 1 = 0 \quad (1)$$

where  $\mathbf{p}$  is the particle position,  $\rho_0$  is the rest density and  $\rho$  is the current density. The density is estimated by standard Smoothed Particle Hydrodynamics (SPH) formulation. We use the Poly6 kernel and Spiky kernel as the smoothing kernel[11].

Since the smoothing kernel has compact support, particles affect only neighbors that are in its kernel radius. Therefore, we set the kernel radius size enough to prevent hair strands penetration when crossing. In our scenes, no penetration has occurred in case of the distance between discrete points is 10(mm), and the kernel radius is 9.3(mm).

#### 3.2. Hair-Air Interaction

##### 3.2.1. Turbulent Flow

In fluid simulation, numerical dissipation damps out various scales of turbulent structures. Fedkiw et al.[2] proposed vorticity confinement to emphasize the exist turbulence for smoke simulation. In PBF, Macklin et al.[1] introduced to use the particles to calculate vorticity confinement. Unlike [1], we apply the energy density  $\hat{e}(s)$  in the spectral band  $s$  at each particle to calculate the corrective force for enhancing the specific scale of turbulence. The vorticity  $\boldsymbol{\omega}$  and the corrective force  $\mathbf{f}$  for the particle  $i$  is calculated as:

$$\begin{aligned} \boldsymbol{\omega}_i &= \nabla \times \mathbf{v} = \sum_j m_j \mathbf{v}_{ij} \times \nabla \mathbf{p}_j W(\mathbf{p}_i - \mathbf{p}_j, h) \\ \mathbf{f}_i &= \epsilon \hat{e}_i(s) (\mathbf{N} \times \boldsymbol{\omega}_i) \end{aligned} \quad (2)$$

where  $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i$  is the difference of the particle velocity  $\mathbf{v}_i, \mathbf{v}_j$ ,  $m$  is the mass,  $W(\mathbf{r}, h)$  is the smoothing kernel with the kernel radius  $h$ ,  $\epsilon$  is a tunable parameter and  $\mathbf{N} = \nabla |\boldsymbol{\omega}_i| / |\nabla |\boldsymbol{\omega}_i||$  is a unit vector that points the peak in the vorticity field. We added this corrective force only to air particles, and the parameter  $\epsilon$  is set 0.008 in our simulations.

In order to calculate  $\hat{e}(s)$ , Kim et al.[12] and Fujisawa et al.[13] used a wavelet transform into the velocity field. We apply the energy density estimator given in [13]:

$$\hat{e}_i(s) = \frac{1}{2} m_i |\hat{\mathbf{v}}_i(s)|^2 \quad (4)$$

$$\hat{\mathbf{v}}_i(s) = \sum_j \mathbf{v}_j \psi(\mathbf{p}_i - \mathbf{p}_j, s) \quad (5)$$

where  $\hat{\mathbf{v}}(s)$  is the spectral component of the velocity field at each particle and  $\psi$  is the mother wavelet. We use a normalized three-dimensional Mexican Hat as the mother wavelet in our simulation:

$$\psi(\mathbf{r}, s) = \frac{2}{\sqrt{15\pi^{\frac{3}{4}} s^{\frac{3}{2}}}} \left( 3 - \frac{|\mathbf{r}|^2}{s^2} \right) \exp\left(-\frac{|\mathbf{r}|^2}{2s^2}\right). \quad (6)$$

Since the influence of distant particles is very small, neighbor particles that are within  $3s$  from the particle  $i$  are considered in Eq.(5).

### 3.2.2. Viscosity with XSPH

To apply viscosity into fluid, we implemented XSPH artificial viscosity[14]. This is easy to tune and stable with high viscosity. Every time step the velocity is updated as:

$$\mathbf{v}_i \leftarrow \mathbf{v}_i + c \sum_j \frac{m_j}{\rho_j} (\mathbf{v}_j - \mathbf{v}_i) \cdot W(\mathbf{p}_i - \mathbf{p}_j, h) \quad (7)$$

$$c = \min(c_i, c_j) \quad (8)$$

where  $c$  is the tunable parameter representing the viscosity property of each particle and  $\min(c_i, c_j)$  is the smaller parameter of the two particles  $i, j$ . In our result, we set  $c_{\text{air}} = 0.014$  for the air particles.

## 4. RESULTS

This section shows the results of applying our approach in several scenes that blow wind (the velocity is  $3(\text{m/s})$ ) from the left side. We implemented the algorithm in NVIDIA CUDA, so that take advantage of parallel computing. All results tested on a computer that equips 4.0 GHz Intel Core i7 6700K CPU and NVIDIA GeForce RTX 2080Ti GPU. In all scenes, we set the number of solver iterations to a fixed value (10 for rod dynamics and 2 for PBF) and the timestep  $\Delta t = 1.38(\text{ms})$ . The hair model has 550 strands (350(mm) length, 35 discretized points) and the number of air particles is 200,000. All particles have the same rest density  $\rho_0 = 0.00129(\text{mg}/\text{mm}^3)$  and the mass  $m = 0.146(\text{mg})$ . Other parameters and performances in each scene are shown in Table 1. All resulting animations are included in our movie file on our website:

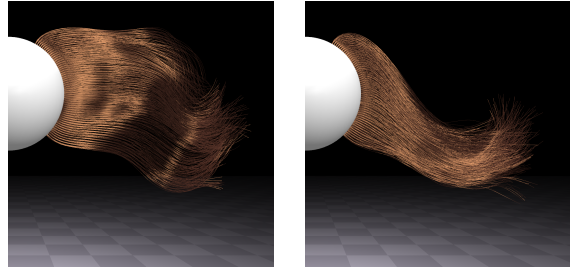
[http://slis.tsukuba.ac.jp/pbcglab/files/ievc\\_hair.mp4](http://slis.tsukuba.ac.jp/pbcglab/files/ievc_hair.mp4)

Fig.2 shows a comparison of the simple hair-air interaction using a lift and drag force model[15] and our method. Lift and drag force model enables fast-computing, and it is suitable for wide and strong air flow like a storm. In contrast, our method can realize the turbulent effect of the air flow. Fig.2(b) shows that the hair forms a circle shape due to the vortex caused by turbulent flow, while the lift and drag force model just make an oscillating motion.

Fig.3 shows straight hair streaming in different parameters. The emphasis of the vortex in the turbulent flow caused different behavior depending on the parameter  $s$ . In addition, the high value of the viscosity parameter of hair  $c_{\text{hair}}$  realizes the gathered hair. Fig.4 shows a streaming of curly hair. Our simulation can naturally treat the interaction between air and curly hair.

Table 1: Parameters and performances of each scene

	scale $s(\text{mm})$	viscosity $c_{\text{hair}}$	time(ms/step)
Fig.2(a)	–	1.4	2.3
Fig.2(b)	6	1.4	10.8
Fig.3(a)	6	1.4	10.8
Fig.3(b)	6	0.05	10.8
Fig.3(c)	18	1.4	71.4
Fig.4	6	1.4	10.8



(a) Lift and drag model

(b) Our method

Figure 2: Comparison with lift and drag force model

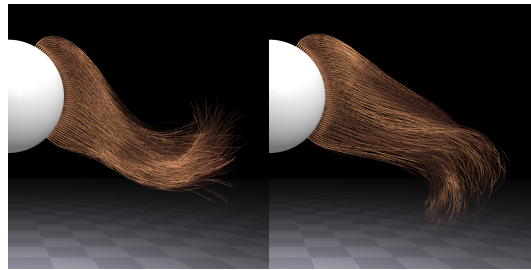
## 5. DISCUSSION AND CONCLUSIONS

We have presented an approach that uses a unified solver to simulate hair-hair and hair-air interaction by treating the hair as a continuum fluid. Moreover, we successfully applied vorticity confinement and wavelet analysis to consider turbulent flow occurring in hair streaming. Our results show that the drastic effects of the turbulent flow and the controllability of vortices. In addition, the difference in the viscosity parameter between hair and air shows another effect in the hair animation.

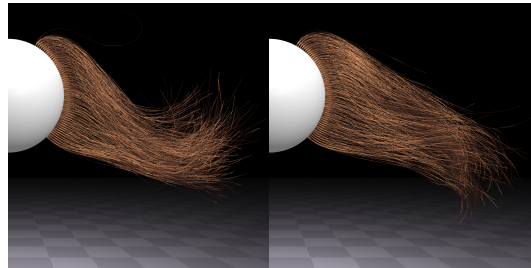
On the other hand, there are some limitations of our presented approach. Because we modeled the hair particles as a sphere shape, it can not represent the fineness of hair fiber (0.08mm)[16]. This makes it difficult to realize the phenomenon that air passes through between each hair strand. Also, our method can treat only one specific scale of exist vortices. Additional control force is necessary for make more intuitive control. Another problem is that our method cannot consider the frictional effects in hair-hair interactions. Our future work includes solving these problems.

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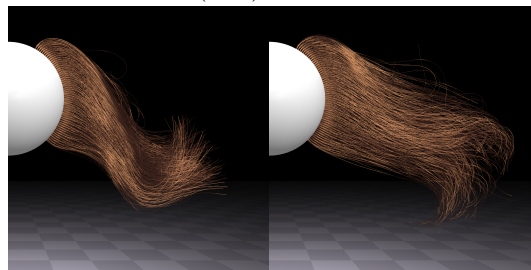
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(a)  $s = 6(\text{mm})$  and  $c_{\text{hair}} = 1.4$



(b)  $s = 6(\text{mm})$  and  $c_{\text{hair}} = 0.05$



(c)  $s = 18(\text{mm})$  and  $c_{\text{hair}} = 1.4$

Figure 3: Straight hair streaming with turbulent flow in different parameters

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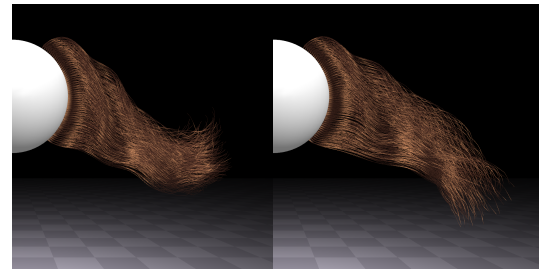


Figure 4: Curly hair streaming with turbulent flow

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